

Competition between α -decay and β -decay for Heavy and Superheavy Nuclei^{*}

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Abstract: In this work, the β -stable region for $Z \geq 90$ is proposed. The calculated β -stable nuclei in the β -stable region are in good agreement with the ones obtained by Möller *et al.*. The half-lives of the nuclei close to the β -stable region are calculated and the competition between α -decay and β -decay is systematically investigated. The calculated half-lives and the suggested decay modes are well in line with the experimental results. The predictions for half-lives and decay modes of the nuclei with $Z = 107-110$ are presented.

Key words: decay, β -stable region, half-life, superheavy nuclei

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1 Introduction

After Becquerel discovered nuclear radioactivity in 1896, scientists began to investigate nuclear decay and found several decay modes. α -decay and β -decay are two of the most important decay modes. α -decay was first explained by Rutherford in 1908 [1]. In the 1930s, Fermi proposed the basic theory of β -decay [2]. From then on, α -decay and β -decay have been widely studied theoretically and experimentally [3–10], and a large number of research results and publications have come out. Generally speaking, α -decay mostly occurs in heavy and superheavy nuclei, while β -decay can occur throughout the whole periodic Table.

In the early stage of the development of nuclear physics, scientists could only study the properties of the nuclei very close to the β -stable line. As a result, many nuclear phenomena, laws, formulae, methods, and models were based on the long-lived nuclei or stable nuclei close to the β -stable line. It is much easier to find and synthesize new nuclei close to the β -stable line. Nowadays, with the development of radioactive nuclear beams, many nuclei far from the β -stable line have been studied. Many new experimental phenomena have been discovered. At present, the stable nuclei with $Z < 83$ are very clear and definite. On the other hand, most nuclei beyond $Z = 83$ are unstable, and their half-lives are usually short. In this article, the β -stable region for $Z \geq 90$

will be proposed. The half-lives of the nuclei close to the β -stable region will be calculated and the competition between α -decay and β -decay will be investigated. Then the decay modes can be suggested by the results of competition.

This article is organized in the following way. In Sec. 2, the β -stable region for $Z \geq 90$ is proposed. In Sec. 3, the half-lives of the nuclei close to the β -stable region are calculated and the competition between α -decay and β -decay is studied. A summary is given in Sec. 4.

2 The β -stable region for $Z \geq 90$

The β -stable line for $Z < 83$ has been well studied by physicists. For heavy and superheavy nuclei with $Z \geq 90$, most of them can occur α -decay and β -decay simultaneously, and their half-lives are usually short. For this reason, it is more important to study the β -stable region than to study the β -stable line for these heavy and superheavy nuclei. In this section, we investigate the boundary of the β -stable region based on a successful binding energy formula.

To accurately measure and calculate the ground-state nuclear binding energies (or masses) is an important goal of nuclear physicists. The binding energy plays a crucial role for the nuclear stability on β -decay, α -decay and spontaneous fission of heavy-mass region with $Z \geq 90$.

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In Ref. [11], Dong and Ren proposed a binding energy formula for heavy and superheavy nuclei. One can accurately reproduce the binding energies for the known heavy and superheavy nuclei with this formula. This formula is useful for accurately estimating the binding energies of unknown superheavy nuclei. Its form is the following:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p A^{-1/2} + \frac{a_6 |A - 252|}{A} - \frac{a_7 |N - 152|}{N} + \frac{a_8 |N - Z - 50|}{A}. \quad (1)$$

The best fit parameters are

$$\begin{cases} a_v = 15.8032 \text{ MeV}, \\ a_s = 17.8147 \text{ MeV}, \\ a_c = 0.71478 \text{ MeV}, \\ a_a = 97.6619 \text{ MeV}, \\ a_6 = 5.33 \text{ MeV}, \\ a_7 = 21.0 \text{ MeV}, \\ a_8 = -15.25 \text{ MeV}. \end{cases} \quad (2)$$

The coefficients of the pairing energy are

$$a_p = \begin{cases} 12.26 \text{ MeV}, & \text{even-even nuclei}, \\ 3.0 \text{ MeV}, & \text{even-odd nuclei}, \\ 0 \text{ MeV}, & \text{odd-even nuclei}, \\ -8.0 \text{ MeV}, & \text{odd-odd nuclei}. \end{cases} \quad (3)$$

The mass formula has the form:

$$\begin{aligned} M(Z, A) &= ZM_H + NM_n - B(Z, A) \\ &= AM_n + Z(M_H - M_n) - B(Z, A), \end{aligned} \quad (4)$$

where $(M_H - M_n) = -0.782 \text{ MeV}$.

The decay energies of β^- -decay and β^+ -decay can be written as:

$$E_d(\beta^-) = M(Z, A) - M(Z + 1, A), \quad (5)$$

$$E_d(\beta^+) = M(Z, A) - M(Z - 1, A) - 2m_e, \quad (6)$$

where $2m_e = 1.022 \text{ MeV}$.

From the Eqs. (1), (4), (5) and (6), we get:

$$\begin{aligned} E_d(\beta^-) &= 0.782 - a_c(2Z + 1)A^{-1/3} + \frac{a_a(A - 2Z - 1)}{A} \\ &\quad + a_7 \left(\frac{|A - Z - 152|}{A - Z} - \frac{|A - Z - 153|}{A - Z - 1} \right) \\ &\quad + \frac{a_8}{A} (|A - 2Z - 52| - |A - 2Z - 50|), \end{aligned} \quad (7)$$

$$\begin{aligned} E_d(\beta^+) &= -1.804 - a_c(-2Z + 1)A^{-1/3} - \frac{a_a(A - 2Z + 1)}{A} \\ &\quad + a_7 \left(\frac{|A - Z - 152|}{A - Z} - \frac{|A - Z - 151|}{A - Z + 1} \right) \\ &\quad + \frac{a_8}{A} (|A - 2Z - 48| - |A - 2Z - 50|). \end{aligned} \quad (8)$$

If the values of $E_d(\beta^-)$ and $E_d(\beta^+)$ are set to zero, one can get the limits of β^- -decay and β^+ -decay for each isotopic chain. For each fixed proton number Z , one can get two different mass numbers for the limits of β^- -decay and β^+ -decay, respectively. For all the proton numbers from $Z = 90$ to $Z = 126$, two sets of mass numbers for the limits of β^- -decay and β^+ -decay can be obtained, respectively. Connecting two sets of mass numbers for the limits of β^- -decay and β^+ -decay in the coordinate space (Z, A) , the boundary of the limits of β^- -decay and β^+ -decay can be obtained. The calculated results are plotted in figure 1.

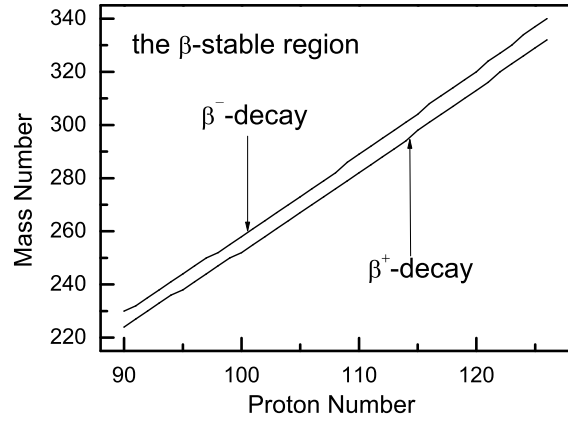


Fig. 1. The calculated β -stable region for $Z \geq 90$. The two curves denote the limits of β^- -decay and β^+ -decay, respectively.

In figure 1, the two curves denote the limits of β^- -decay and β^+ -decay, respectively. They are almost parallel. The calculated β -stable region is a long and narrow region between the two curves. According to the calculations, the nuclei above the calculated β -stable region can occur β^- -decay and the nuclei below the region can occur β^+ -decay. The nuclei in the β -stable region are the possible β -stable nuclei. Because there are few experimental data, we compare our calculated results with the results given by Möller *et al.* [12]. The comparison between our calculated results and Möller's results are shown in table 1.

Table 1. The possible β -stable nuclei in the calculated β -stable region for $Z \geq 90$. The corresponding β -stable nuclei calculated by Möller *et al.* [12] are listed for comparison.

Z	Mass number A of β -stable nuclei	
	Cal.	Möller
90	224–230	224, 226–230, 232
91	227–232	231
92	230–235	230, 232–236, 238
93	233–238	237
94	236–241	236, 238–242, 244
95	238–244	241#, 243#
96	241–247	240, 242–246, 248
97	244–250	247
98	247–252	246, 248–252, 254
99	250–255	253
100	252–258	252, 254–258, 260, 262
101	255–261	259
102	258–264	258, 260–264, 266
103	261–267	265
104	264–270	264, 266–268, 270, 272, 274
105	267–273	269, 271
106	270–276	268, 270, 272–276, 278, 280
107	273–279	277
108	276–282	274, 276, 278–284, 286
109	279–286	283#, 285#
110	282–289	282, 284–288, 290
111	285–292	289
112	288–295	288, 290–294, 296
113	291–298	293#, 295#
114	294–301	292, 294–298, 300, 302, 304
115	298–304	299, 301, 303
116	301–308	300, 302, 304–306, 308, 310–312, 314
117	304–311	307, 309
118	307–314	304, 306, 308, 310–314, 318
119	310–317	315
120	313–320	312, 316–318, 320, 322, 324
121	316–324	319
122	320–327	318, 320, 321, 323–326, 328, 330
123	324–330	322, 327
124	327–334	317, 323, 324, 326, 328–332, 334, 336, 338
125	329–337	325, 327, 333
126	332–340	326, 330, 332, 334–338

In table 1, the calculated possible β -stable nuclei and the results calculated by Möller *et al.* [12] are listed in the second and the third columns, respectively. The mass numbers with # denote that the nuclei with these mass numbers are β -stable nuclei by estimated from systematic trends in neighboring nuclei. Our calculated results show that there are several (from six to nine) β -stable nuclei in each isotopic chain and they are continuous in their isotopic chain. For even Z, the two results are almost the same. On the whole, the range by Möller *et al.* is slightly larger than our calculated results. The calculated β -stable nuclei by Möller *et al.* are not continuous in their isotopic chains. Some nuclei are β -stable nuclei from our calculations, but they are not β -stable nuclei from Möller's results, and vice versa. For odd Z, Möller's results show that there are only one or two β -stable nuclei in their isotopic chains except for Z = 115,

125. It is different from our results. But it can be seen that the β -stable nuclei from Möller's results are all in the middle of our calculated β -stable region except for Z = 123, 125. From the above discussions, it can be said that the calculated β -stable region are in good agreement with the Möller's results. For further comprehending the calculated β -stable region, the β -stable nuclei from both the calculated results and Möller's results are drawn in figure 2.

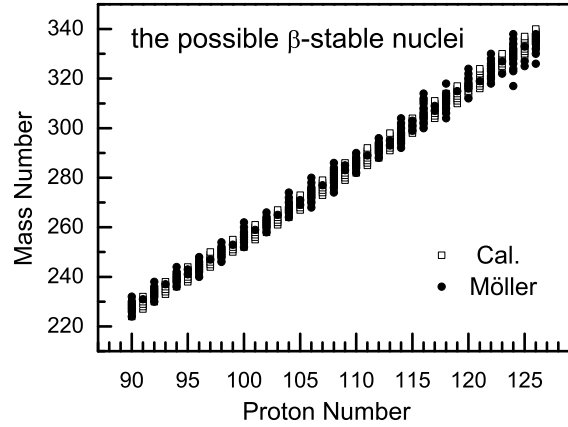


Fig. 2. The possible β -stable nuclei for $Z \geq 90$. The corresponding β -stable nuclei calculated by Möller *et al.* [12] are listed for comparison.

In figure 2, the hollow squares denote the possible β -stable nuclei obtained by our calculations, and the dark circles are the ones from Möller's results. For even Z, the β -stable nuclei obtained from Möller's results almost cover the calculated β -stable nuclei. For odd Z, except Z = 123, 125, the β -stable nuclei obtained from Möller's results are included in the calculated β -stable nuclei. It is in line with the above discussions.

3 Competition between α -decay and β -decay of the nuclei close to the β -stable region

In the previous section, the β -stable region for $Z \geq 90$ has been proposed. Most nuclei with $Z \geq 90$ can occur α -decay, β -decay and spontaneous fission simultaneously. In this section, we will calculate the half-lives of the nuclei close to the calculated β -stable region, and study the competition between α -decay and β -decay of them. It is a very interesting topic. There are plenty of experimental half-lives and the decay modes of many nuclei are very explicit in this region. The calculated results can be compared with these experimental data and the reliability of the calculated results can be tested. On the other hand, the predictions are useful for quickly estimating

the decay modes and half-lives of future superheavy experiments. Before calculating the half-lives, we firstly introduce several successful formulae for calculating.

α -decay is a very general decay mode for the ground states of heavy and superheavy nuclei. In Ref. [13], Ni *et al.* proposed a unified formula of half-lives for α -decay and cluster radioactivity. For α -decay, it is written as:

$$\log_{10} T_{1/2} = 2a\sqrt{\mu}(Z-2)Q_{\alpha}^{-1/2} + b\sqrt{\mu}[2(Z-2)]^{1/2} + c, \quad (9)$$

where $\mu = 4(A-4)/A$, $T_{1/2}$ is the half-life of α -decay (in seconds), and Q_{α} is α -decay energy (in MeV). A and Z are the mass number and the proton number of the parent nuclei respectively. The values of the parameters are $a = 0.39961$, $b = -1.31008$. Parameter c is determined to be $c_{e-e} = -17.00698$ (for even-even nuclei), $c_{e-o} = -16.26029$ (for even-odd nuclei), $c_{o-e} = -16.40484$ (for odd-even nuclei), and $c_{o-o} = -15.85337$ (for odd-odd nuclei).

β -decay is also a very important decay mode for rich-neutron or rich-proton nuclei. For β^{-} -decay, in Ref. [14], Zhang *et al.* proposed a reliable formula to calculate the β^{-} -decay half-lives. It is written as:

$$\log_{10} T_{1/2} = (c_1 Z + c_2)N + c_3 Z + c_4 + \text{shell}(Z, N), \quad (10)$$

where

$$\begin{aligned} \text{shell}(Z, N) = & c_5 [e^{-(N-29)^2/15} + e^{-(N-50)^2/37} \\ & + e^{-(N-85)^2/9} + e^{-(N-131)^2/3}] \\ & + c_6 e^{-[(Z-51.5)^2 + (N-80.5)^2]/1.9} \end{aligned} \quad (11)$$

is the shell correction term. Z and N are the proton number and neutron number of the parent nuclei. $T_{1/2}$ is the half-life of β^{-} -decay (in seconds). The parameters are $c_1 = 3.37 \times 10^{-4}$, $c_2 = -0.2558$, $c_3 = 0.4028$, $c_4 = -1.0100$, $c_5 = 0.9039$, and $c_6 = 7.7139$.

For β^{+} -decay, in Ref. [15], Zhang *et al.* proposed a similar formula to the Eq. (10). It is written as:

$$\log_{10} T_{1/2} = (c_1 Z + c_2)N + c_3 Z + c_4. \quad (12)$$

For different order (the allowed β^{+} -transition, the first and the second forbidden β^{+} -transition), the parameters are different. The even-odd effect has been taken into account in the above equation. The best fit parameters are displayed in table 2.

Table 2. The parameters of the Eq. (12). The word “order” in the first column denotes the order of the β^{+} -decay from ground state to ground state. The even-odd effect has been included.

order	c_1	c_2	c_3	c_4		
				e-o, o-e	o-o	e-e
allowed	-0.00179	0.4233	-0.3405	-0.6443	-1.7089	-0.2132
first	-0.00127	0.3992	-0.4183	3.8215	3.7969	4.0364
second	-0.00162	0.3980	-0.3286	-0.1618	-0.4854	0.0267

For a given proton number Z , we select ten continuous isotopes nearest to the top and bottom of the β -stable region, respectively. Thus there are 20 nuclei for each isotopic chain. Because only the half-lives of the allowed β^{+} -transition, the first and the second forbidden β^{+} -transition can be calculated by the Eq. (12), the nuclei with higher forbidden β^{+} -transition are not included. Because the formula (9) can only calculate the half-lives of the nuclei with $Z \geq 84$ and $N \geq 128$, the nuclei with $N < 128$ are not included also. So the num-

ber of the calculated nuclei of each isotopic chain may be less than 20. We calculate the half-lives of the nuclei from $Z = 90$ to $Z = 126$, and predict the decay modes of them. Because the calculated data are too many, we firstly compare the calculated results with the available experimental data [16]. The selected region for comparison is from $Z = 90$ to $Z = 103$, because there are many experimental data in this region. The results are listed in table 3.

Table 3: The comparison of the half-lives and decay modes between the calculated results and the experimental data by Audi *et al.* [16] from $Z = 90$ to $Z = 103$. Here $C = \log_{10}(T_{1/2}^{cal}/T_{1/2}^{expt})$.

Z	A	Cal. $T_{\alpha 1/2}^{cal}$ (s)	Cal. $T_{\beta 1/2}^{cal}$ (s)	Calculated decay modes and intensities(%)	Experimental decay modes and intensities(%)	$T_{1/2}^{expt}$ (s)	C
90	218	9.14×10^{-8}	718.8	$\alpha = 100$	$\alpha = 100$	1.09×10^{-7}	-0.08
90	219	3.11×10^{-6}	844.5	$\alpha = 100$	$\alpha = 100$	1.05×10^{-6}	0.47
90	221	5.67×10^{-4}	3136.2	$\alpha = 100$	$\alpha = 100$	1.68×10^{-3}	-0.47
90	222	0.0028	9912.9	$\alpha = 100$	$\alpha = 100$	2.05×10^{-3}	0.14

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Table 3 – continued from previous page

Z	A	Cal. $T_{\alpha 1/2}^{cal}$ (s)	Cal. $T_{\beta 1/2}^{cal}$ (s)	Calculated decay modes and intensities(%)	Experimental decay modes and intensities(%)	$T_{1/2}^{expt}$ (s)	C
90	223	0.93	1.16×10^4	$\alpha = 100$	$\alpha = 100$	0.6	0.19
90	231	1.63×10^{17}	2823.1	$\beta^- = 100$	$\beta^- = 100$	9.19×10^4	-1.51
90	233	2.13×10^{21}	999.5	$\beta^- = 100$	$\beta^- = 100$	1338	-0.13
90	234	2.02×10^{21}	594.7	$\beta^- = 100$	$\beta^- = 100$	2.08×10^6	-3.54
90	235	3.78×10^{24}	353.9	$\beta^- = 100$	$\beta^- = 100$	432	-0.09
90	236	2.44×10^{24}	210.6	$\beta^- = 100$	$\beta^- = 100$	2250	-1.03
90	237	4.61×10^{29}	125.3	$\beta^- = 100$	$\beta^- = 100$	288	-0.36
90	238	2.87×10^{29}	74.6	$\beta^- = 100$	$\beta^- = 100$	564	-0.88
91	219	2.27×10^{-7}	115.0	$\alpha = 100$	$\alpha = 100$	5.30×10^{-8}	0.63
91	220	2.94×10^{-6}	208.9	$\alpha = 100$	$\alpha = 100$	7.80×10^{-7}	0.58
91	221	2.02×10^{-5}	424.7	$\alpha = 100$	$\alpha = 100$	5.90×10^{-6}	0.53
91	223	0.0063	1568.1	$\alpha = 100$	$\alpha = 100, \beta^+ < 0.001\#$	5.10×10^{-3}	0.09
91	225	6.5	5789.2	$\alpha = 100$	$\alpha = 100$	1.7	0.58
91	226	694.0	1.05×10^4	$\alpha = 94, \beta^+ = 6$	$\alpha = 74, \beta^+ = 26$	108	0.78
91	233	7.47×10^{16}	4741.5	$\beta^- = 100$	$\beta^- = 100$	2.33×10^6	-2.69
91	234	1.15×10^{19}	2823.4	$\beta^- = 100$	$\beta^- = 100$	2.41×10^4	-0.93
91	235	3.53×10^{19}	1681.3	$\beta^- = 100$	$\beta^- = 100$	1470	0.06
91	236	1.35×10^{22}	1001.2	$\beta^- = 100$	$\beta^- = 100$	546	0.26
91	237	1.17×10^{22}	596.2	$\beta^- = 100$	$\beta^- = 100$	522	0.06
91	238	7.10×10^{27}	355.0	$\beta^- = 100$	$\beta^- = 100$	136	0.42
91	239	8.19×10^{27}	211.4	$\beta^- = 100$	$\beta^- = 100$	6.48×10^3	-1.49
92	223	3.83×10^{-4}	212.4	$\alpha = 100$	$\alpha \approx 100, \beta^+ = 0.2\#$	2.10×10^{-5}	1.26
92	225	0.18	779.4	$\alpha = 100$	$\alpha = 100$	0.061	0.47
92	226	0.34	2449.3	$\alpha = 100$	$\alpha = 100$	0.269	0.10
92	227	99.1	2860.9	$\alpha = 97, \beta^+ = 3$	$\alpha = 100, \beta^+ < 0.001\#$	66	0.16
92	228	659.4	4508.8	$\alpha = 87, \beta^+ = 13$	$\alpha > 95, \varepsilon < 5$	546	0.02
92	229	8.59×10^4	2888.3	$\alpha = 3, \beta^+ = 97$	$\alpha \approx 20, \beta^+ \approx 80$	3.48×10^3	-0.10
92	236	3.37×10^{15}	4753.1	$\beta^- = 100$	$\alpha = 100$	7.39×10^{14}	
92	237	1.62×10^{18}	2832.6	$\beta^- = 100$	$\beta^- = 100$	5.83×10^5	-2.30
92	239	1.62×10^{21}	1006.0	$\beta^- = 100$	$\beta^- = 100$	1.41×10^3	-0.15
92	240	8.57×10^{20}	599.5	$\beta^- = 100$	$\beta^- = 100$	5.08×10^4	-1.92
92	242	4.35×10^{23}	212.9	$\beta^- = 100$	$\beta^- = 100$	1.01×10^3	-0.68
93	227	1.34	385.2	$\alpha = 100$	$\alpha \approx 100, \beta^+ = 0.05\#$	0.51	0.42
93	228	118.8	695.3	$\alpha = 85, \beta^+ = 15$	$\alpha = 40, \varepsilon = 60$	61.4	0.22
93	229	1033.1	1405.5	$\alpha = 58, \beta^+ = 42$	$\alpha > 50, \beta^+ < 50$	240	0.39
93	230	3.00×10^4	64.6	$\beta^+ = 100$	$\beta^+ \leq 97, \alpha \geq 3$	276	-0.63
93	231	4.85×10^5	1354.4	$\beta^+ = 100$	$\beta^+ = 98, \alpha = 2$	2.93×10^3	-0.34
93	239	1.23×10^{16}	4779.5	$\beta^- = 100$	$\beta^- = 100$	2.04×10^5	-1.63
93	240	6.17×10^{16}	2850.5	$\beta^- = 100$	$\beta^- = 100$	3.71×10^3	-0.11
93	241	8.45×10^{18}	1700.1	$\beta^- = 100$	$\beta^- = 100$	834	0.31
93	242	2.05×10^{19}	1013.9	$\beta^- = 100$	$\beta^- = 100$	132	0.86
93	243	4.96×10^{19}	604.7	$\beta^- = 100$	$\beta^- = 100$	111	0.74
93	244	3.11×10^{21}	360.7	$\beta^- = 100$	$\beta^- = 100$	137	0.42
94	229	24.3	189.2	$\alpha = 89, \beta^+ = 11$	$\alpha = 100$	120	
94	231	1.78×10^4	194.6	$\alpha = 1, \beta^+ = 99$	$\alpha = 13, \beta^+ = 87$	516	-0.43
94	233	1.10×10^6	629.9	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha = 0.12$	1.25×10^3	-0.30
94	235	1.86×10^8	2038.6	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha = 0.003$	1.52×10^3	0.13
94	242	1.17×10^{13}	4821.1	$\beta^- = 100$	$\alpha = 100$	4.93×10^{11}	
94	243	2.30×10^{15}	2877.5	$\beta^- = 100$	$\beta^- = 100$	1.78×10^4	-0.79
94	244	1.85×10^{15}	1717.5	$\beta^- = 100$	$\alpha \approx 100, \text{SF} = 0.12$	1.05×10^{14}	
94	245	2.50×10^{17}	1025.1	$\beta^- = 100$	$\beta^- = 100$	3.78×10^4	-1.57
94	246	4.72×10^{17}	611.9	$\beta^- = 100$	$\beta^- = 100$	9.36×10^5	-3.18

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Table 3 – continued from previous page

Z	A	Cal. $T_{\alpha 1/2}^{cal}$ (s)	Cal. $T_{\beta 1/2}^{cal}$ (s)	Calculated decay modes and intensities(%)	Experimental decay modes and intensities(%)	$T_{1/2}^{expt}$ (s)	C
94	247	1.00×10^{20}	365.2	$\beta^- = 100$	$\beta^- = 100$	1.96×10^5	-2.73
95	235	3.10×10^5	1202.3	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha = 0.4$	594	0.31
95	237	2.29×10^7	932.6	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha = 0.025$	4.38×10^3	-0.67
95	245	5.58×10^{12}	4878.1	$\beta^- = 100$	$\beta^- = 100$	7.38×10^3	-0.18
95	246	4.73×10^{13}	2913.8	$\beta^- = 100$	$\beta^- = 100$	2.34×10^3	0.10
95	247	1.27×10^{15}	1740.5	$\beta^- = 100$	$\beta^- = 100$	1.38×10^3	0.10
96	239	1.58×10^6	927.3	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha < 0.1$	1.04×10^4	-1.05
96	249	2.21×10^{13}	2959.7	$\beta^- = 100$	$\beta^- = 100$	3.85×10^3	-0.11
96	251	1.89×10^{13}	1057.7	$\beta^- = 100$	$\beta^- = 100$	1.01×10^3	0.02
97	242	5.02×10^5	1750.9	$\beta^+ = 100$	$\beta^+ \approx 100$	420	0.62
97	243	1.65×10^5	3498.3	$\alpha = 2, \beta^+ = 98$	$\alpha \approx 0.15, \beta^+ \approx 100$	1.62×10^4	-0.67
97	251	1.38×10^{11}	5040.9	$\beta^- = 100$	$\beta^- = 100$	3.34×10^3	0.18
98	241	522.5	130.6	$\alpha = 20, \beta^+ = 80$	$\alpha \approx 25, \beta^+ \approx 75$	228	-0.34
98	243	8779.7	462.8	$\alpha = 5, \beta^+ = 95$	$\alpha \approx 16, \beta^+ \approx 84$	642	-0.17
98	245	1.66×10^4	1640.1	$\alpha = 9, \beta^+ = 91$	$\alpha = 36, \beta^+ = 64$	2.70×10^3	-0.26
98	246	1.22×10^5	5064.3	$\alpha = 4, \beta^+ = 96$	$\alpha = 100$	1.28×10^5	
98	253	1.69×10^9	8598.6	$\beta^- = 100$	$\beta^- \approx 100, \alpha \approx 0.31$	1.54×10^6	-2.25
98	255	1.84×10^{11}	3082.3	$\beta^- = 100$	$\beta^- = 100$	5.10×10^3	-0.22
99	243	33.4	61.6	$\alpha = 65, \beta^+ = 35$	$\alpha \geq 30, \beta^+ \leq 70$	21	0.01
99	245	120.3	217.0	$\alpha = 64, \beta^+ = 36$	$\alpha = 40, \beta^+ = 60$	66	0.07
99	247	3892.3	116.8	$\alpha = 3, \beta^+ = 97$	$\alpha \approx 7, \beta^+ \approx 93$	276	-0.39
99	248	2.65×10^5	1355.9	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha \approx 0.25$	1620	-0.08
99	249	5.97×10^5	2693.4	$\beta^+ = 100$	$\beta^+ \approx 100, \alpha \approx 0.57$	6.13×10^3	-0.36
99	256	3.72×10^9	8802.2	$\beta^- = 100$	$\beta^- = 100$	1.52×10^3	0.76
99	257	8.50×10^9	5274.1	$\beta^- = 100$	$\beta^- = 100$	6.65×10^5	-2.10
100	247	37.2	16.5	$\alpha = 31, \beta^+ = 69$	$\alpha \geq 50, \beta^+ \leq 50$	35	-0.49
100	248	34.3	310.5	$\alpha = 90, \beta^+ = 10$	$\alpha = 93, \beta^+ = 7$	36	-0.07
100	251	2.39×10^4	411.3	$\alpha = 2, \beta^+ = 98$	$\alpha = 1.8, \beta^+ = 98.2$	1.91×10^4	-1.67
101	250	106.6	82.6	$\alpha = 44, \beta^+ = 56$	$\alpha = 7, \beta^+ = 93$	52	-0.05
101	252	2655.3	3.30	$\beta^+ = 100$	$\beta^+ > 50, \alpha < 50$	138	
102	254	33.3	228.1	$\alpha = 87, \beta^+ = 13$	$\alpha = 90, \beta^+ = 10$	51	-0.25
102	256	2.33	789.7	$\alpha = 100$	$\alpha \approx 100$	2.91	-0.10
103	253	1.68	1.30	$\alpha = 44, \beta^+ = 56$	$\alpha = 90, \beta^+ = 1\#$	0.58	-0.01
103	254	16.8	17.3	$\alpha = 51, \beta^+ = 49$	$\alpha = 76, \beta^+ = 24$	13	-0.19
103	256	13.6	0.60	$\alpha = 4, \beta^+ = 96$	$\alpha = 85, \beta^+ = 15$	27	-1.68

The calculated half-lives are obtained from the formulae (9), (10) and (12). When using the formula (9) to calculate half-lives of α -decay and β^+ -decay, we need α -decay energies Q_α , spins and parities. Here, All the values of Q_α are taken from Ref. [16, 17]. If there are no experimental data, we use the calculated data obtained by Möller *et al.* [12]. The calculated α -decay half-lives and the calculated β -decay (including β^- -decay and β^+ -decay) half-lives (in seconds) are listed in the third and the fourth columns in table 3. The fifth column is the calculated decay modes and intensities (in %). The decay mode can be regard as a competition between α -decay and β -decay. Here we define a symbol R to denote the ratio of α -decay half-life and β -decay half-life. It is defined as: $R = T_{\alpha 1/2}^{cal}/T_{\beta 1/2}^{cal}$. If the α -decay half-life is shorter than the β -decay half-life by 100 times (i.e. $R < 0.01$)

in a nucleus, we can say the decay mode of this nucleus is α -decay. If the α -decay half-life is longer than the β -decay half-life by 100 times (i.e. $R > 100$) in a nucleus, we can say the decay mode of this nucleus is β -decay. If $0.01 < R < 100$, the decay mode can be regard as a coexistence state of both α -decay and β -decay, and we use the symbol $\alpha + \beta^+$ to denote the coexistence state of both α -decay and β^+ -decay. The sixth and seventh columns are the experimental decay modes and intensities (in %) and half-lives. The data marked with # denote the values from systematic trends in neighboring nuclei. The symbol C in the last column is the ratio of calculated half-life and experimental one, and it is in the form of $C = \log_{10}(T_{1/2}^{cal}/T_{1/2}^{expt})$. The nuclei which do not have explicit experimental decay modes and intensities (in %) are not included in table 3. There are alto-

gether 91 nuclei. It can be seen that the predicted decay modes are in excellent agreement with the experimental ones. There are only six nuclei whose decay modes are not in line with the predicted decay modes. The proton numbers and the mass numbers of these six nuclei are marked in bold italic type in table 3. It indicates that the predicted decay modes in this region are reliable. For the ratios of calculated half-lives and experimental ones, the values of C are mostly between 1.0 and -1.0 . This means that the most differences between calculated half-lives and experimental ones are less than 10 times. It is a good approximation. Usually, if the differences between theoretical half-lives and experimental ones are less than 10^3 times, it can be said that the results are satisfactory. So we can say the calculated half-lives in this region are in good agreement with the experimental data.

After having compared the calculated results with the

experimental data from $Z = 90$ to $Z = 103$, we predict some half-lives and decay modes for some heavier proton number Z . Synthesizing superheavy nuclei is a hot topic of nuclear physics. At present, Chinese physicists are endeavoring to synthesize some superheavy nuclei around $Z = 110$, and they have obtained some successful results [18]. Next, we select the region from $Z = 107$ to $Z = 110$ to make some predictions. There are many researches in this region [18–27]. We hope that our predictions will be useful for future experiments on heavy and superheavy nuclei. Because there are almost no explicit spins and parities for the nuclei with $Z \geq 107$ except even-even nuclei, it is difficult for us to judge the orders of β^+ -decay. When using the formula (12) to calculate half-lives of β^+ -decay, we suppose all the orders of β^+ -decay are the first β^+ -transition for simplifying the calculations. The calculated results are listed in table 4.

Table 4: The calculated half-lives and the predicted decay modes of the nuclei from $Z = 107$ to $Z = 110$. Some available experimental half-lives of α -decay [18, 19, 22, 27] are listed for comparison. Here $D = T_{\alpha 1/2}^{cal}/T_{\alpha 1/2}^{expt}$.

Z	A	Cal. $T_{\alpha 1/2}^{cal}$ (s)	Cal. $T_{\beta 1/2}^{cal}$ (s)	Calculated decay modes and intensities(%)	$T_{\alpha 1/2}^{expt}$ (s)	D
107	264	0.16	2.40	$\alpha = 94, \beta^+ = 6$	0.9 [27]	0.18
107	265	1.99	4.64	$\alpha = 70, \beta^+ = 30$	0.94 [18]	2.12
107	266	20.9	8.04	$\alpha = 28, \beta^+ = 72$	5 [16]	4.18
107	267	36.3	15.6	$\alpha = 30, \beta^+ = 70$	17 [22]	2.14
107	268	55.3	27	$\alpha = 33, \beta^+ = 67$		
107	269	86.5	52.4	$\alpha = 38, \beta^+ = 62$		
107	270	63.7	90.9	$\alpha = 59, \beta^+ = 41$	61 [19]	1.04
107	271	0.91	176.3	$\alpha = 100$		
107	272	34.1	305.5	$\alpha = 90, \beta^+ = 10$	9.8 [19]	3.48
107	273	56.2	β -stable	$\alpha = 100$		
107	274	3.99×10^3	β -stable	$\alpha = 100$		
107	275	108.5	β -stable	$\alpha = 100$		
107	276	4.00×10^4	β -stable	$\alpha = 100$		
107	277	4.58×10^6	β -stable	$\alpha = 100$		
107	278	9.12×10^5	β -stable	$\alpha = 100$		
107	279	1.07×10^7	β -stable	$\alpha = 100$		
107	280	2.49×10^{10}	1.19×10^4	$\beta^- = 100$		
107	281	3.24×10^{11}	7.16×10^3	$\beta^- = 100$		
107	282	3.42×10^{13}	4.31×10^3	$\beta^- = 100$		
107	283	1.52×10^{12}	2.60×10^3	$\beta^- = 100$		
107	284	1.16×10^{12}	1.57×10^3	$\beta^- = 100$		
107	285	9.45×10^{11}	945.6	$\beta^- = 100$		
107	286	4.41×10^{15}	570.1	$\beta^- = 100$		
107	287	5.48×10^{14}	343.7	$\beta^- = 100$		
107	288	7.69×10^{15}	207.2	$\beta^- = 100$		
107	289	1.43×10^{15}	124.9	$\beta^- = 100$		
108	266	2.72×10^{-3}	1.83	$\alpha = 100$	2.3×10^{-3} [22]	1.18
108	267	0.129	2.04	$\alpha = 94, \beta^+ = 6$	0.058 [22]	2.22
108	268	0.0376	6.12	$\alpha = 100$		
108	269	9.44	6.82	$\alpha = 42, \beta^+ = 58$	9.7 [22]	0.97
108	270	1.88	20.4	$\alpha = 92, \beta^+ = 8$	3.6 [22]	0.52

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Table 4 – continued from previous page

Z	A	Cal. $T_{\alpha 1/2}^{cal}$ (s)	Cal. $T_{\beta 1/2}^{cal}$ (s)	Calculated decay modes and intensities(%)	$T_{\alpha 1/2}^{expt}$ (s)	D
108	271	0.21	22.8	$\alpha = 100$		
108	272	0.011	68.3	$\alpha = 100$		
108	273	0.21	76.2	$\alpha = 100$		
108	274	0.49	228.4	$\alpha = 100$		
108	275	0.31	254.6	$\alpha = 98, \beta^+ = 2$	0.19 [19]	1.63
108	276	66.3	β -stable	$\alpha = 100$		
108	277	8.00×10^3	β -stable	$\alpha = 100$		
108	278	516.2	β -stable	$\alpha = 100$		
108	279	9.31×10^4	β -stable	$\alpha = 100$		
108	280	1.98×10^6	β -stable	$\alpha = 100$		
108	281	4.57×10^{10}	β -stable	$\alpha = 100$		
108	282	1.60×10^{10}	β -stable	$\alpha = 100$		
108	283	3.50×10^{11}	1.25×10^4	$\beta^- = 100$		
108	284	1.60×10^{10}	7.54×10^3	$\beta^- = 100$		
108	285	1.71×10^{10}	4.55×10^3	$\beta^- = 100$		
108	286	8.25×10^9	2.48×10^3	$\beta^- = 100$		
108	287	9.52×10^{13}	1.66×10^3	$\beta^- = 100$		
108	288	4.23×10^{12}	999.3	$\beta^- = 100$		
108	289	1.09×10^{14}	602.9	$\beta^- = 100$		
108	290	2.22×10^{13}	363.8	$\beta^- = 100$		
108	291	1.09×10^{14}	219.5	$\beta^- = 100$		
108	292	4.83×10^{12}	132.5	$\beta^- = 100$		
109	269	7.33×10^{-3}	0.891	$\alpha = 100$		
109	270	3.62×10^{-4}	1.54	$\alpha = 88, \beta^+ = 12$	5×10^{-3} [22]	0.07
109	271	0.073	2.96	$\alpha = 98, \beta^+ = 2$		
109	272	0.018	5.1	$\alpha = 100$		
109	273	1.45×10^{-3}	9.84	$\alpha = 100$		
109	274	1.14	17	$\alpha = 94, \beta^+ = 6$	0.445 [19]	2.56
109	275	9.82×10^{-3}	32.7	$\alpha = 100$	9.7×10^{-3} [19]	1.01
109	276	1.57	56.3	$\alpha = 97, \beta^+ = 3$	0.72 [19]	2.18
109	277	4.28	108.9	$\alpha = 96, \beta^+ = 4$		
109	278	240.6	187.2	$\alpha = 44, \beta^+ = 56$		
109	279	1.29×10^3	β -stable	$\alpha = 100$		
109	280	3.21×10^4	β -stable	$\alpha = 100$		
109	281	1.90×10^5	β -stable	$\alpha = 100$		
109	282	1.94×10^9	β -stable	$\alpha = 100$		
109	283	2.06×10^9	β -stable	$\alpha = 100$		
109	284	1.70×10^{10}	β -stable	$\alpha = 100$		
109	285	8.22×10^8	β -stable	$\alpha = 100$		
109	286	4.41×10^8	β -stable	$\alpha = 100$		
109	287	1.69×10^9	7.97×10^3	$\beta^- = 100$		
109	288	1.40×10^{12}	4.81×10^3	$\beta^- = 100$		
109	289	2.02×10^{12}	2.90×10^3	$\beta^- = 100$		
109	290	1.47×10^{13}	1.75×10^3	$\beta^- = 100$		
109	291	7.61×10^{12}	1.06×10^3	$\beta^- = 100$		
109	292	1.88×10^{13}	639.6	$\beta^- = 100$		
109	293	1.00×10^{12}	386.3	$\beta^- = 100$		
109	294	1.62×10^9	233.2	$\beta^- = 100$		
109	295	9.23×10^8	140.8	$\beta^- = 100$		
109	296	1.40×10^{10}	85.1	$\beta^- = 100$		
110	272	1.02×10^{-4}	1.15	$\alpha = 100$		
110	273	2.16×10^{-4}	1.28	$\alpha = 100$	1.7×10^{-4} [22]	1.27

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Table 4 – continued from previous page

Z	A	Cal. $T_{\alpha 1/2}^{cal}$ (s)	Cal. $T_{\beta 1/2}^{cal}$ (s)	Calculated decay modes and intensities(%)	$T_{\alpha 1/2}^{expt}$ (s)	D
110	274	3.32×10^{-5}	3.81	$\alpha = 100$		
110	275	8.90×10^{-4}	4.23	$\alpha = 100$		
110	276	2.53×10^{-3}	12.6	$\alpha = 100$		
110	277	0.081	14.0	$\alpha = 100$		
110	278	0.091	41.6	$\alpha = 100$		
110	279	1.40	46.1	$\alpha = 97, \beta^+ = 3$	2 [19]	0.7
110	280	9.18	137.5	$\alpha = 94, \beta^+ = 6$		
110	281	584.3	152.4	$\alpha = 21, \beta^+ = 79$	240 [16]	2.43
110	282	1.94×10^9	β -stable	$\alpha = 100$		
110	283	3.97×10^9	β -stable	$\alpha = 100$		
110	284	1.14×10^9	β -stable	$\alpha = 100$		
110	285	1.13×10^9	β -stable	$\alpha = 100$		
110	286	8.62×10^9	β -stable	$\alpha = 100$		
110	287	5.29×10^9	β -stable	$\alpha = 100$		
110	288	8.76×10^9	β -stable	$\alpha = 100$		
110	289	3.36×10^9	β -stable	$\alpha = 100$		
110	290	3.12×10^{10}	8.45×10^3	$\beta^- = 100$		
110	291	1.74×10^{11}	5.10×10^3	$\beta^- = 100$		
110	292	2.24×10^{10}	3.08×10^3	$\beta^- = 100$		
110	293	6.49×10^{10}	1.86×10^3	$\beta^- = 100$		
110	294	6.07×10^9	1.13×10^3	$\beta^- = 100$		
110	295	7.67×10^6	680.7	$\beta^- = 100$		
110	296	6.59×10^6	411.4	$\beta^- = 100$		
110	297	7.84×10^7	248.6	$\beta^- = 100$		
110	298	2.38×10^6	150.2	$\beta^- = 100$		
110	299	2.26×10^8	90.8	$\beta^- = 100$		

In table 4, the calculated half-lives and the suggested decay modes of the nuclei from $Z = 107$ to $Z = 110$ can be clearly seen. The available experimental half-lives of α -decay are listed in the sixth column, and the corresponding references are also listed there. The symbol D in the last column is the ratio of calculated half-life of α -decay and experimental one, and it is in the form of $D = T_{\alpha 1/2}^{cal}/T_{\alpha 1/2}^{expt}$.

For β^- -decay, the values of half-life vary from 10^1 s to 10^5 s for all Z . The nearer the nuclei are close to the β -stable region, the longer their half-lives are. For β^+ -decay, on the whole, it is similar to the case of β^- -decay. For α -decay, the half-lives of α -decay approximately vary from 10^{-4} s to 10^{16} s in this region. The half-life of α -decay of a nucleus above the β -stable region is much longer than its half-life of β^- -decay on the whole. Thus the nuclei above the β -stable region occur β^- -decay mainly. However, for most nuclei below the β -stable region, their half-lives of α -decay are slightly less than the ones of β -decay. So the decay modes of these nuclei are mainly α or $\alpha + \beta^+$ -decay. There are 18 experimental half-lives of α -decay in this region. It can be seen that the calculated half-lives of α -decay are in agreement with the experimental ones. Except for ^{270}Mt

($D = 0.07$), the values of D vary from 0.18 to 4.18. It is a good approximation. It must be pointed out that the half-life is very sensitive to the α -decay energy. A small change in α -decay energy will lead to a very large difference in half-life. There are few experimental α -decay energies in this region, and most α -decay energies used for calculation are the estimated data [17] or the calculated results [12].

To clearly understand the competition between α -decay and β -decay of the nuclei close to the calculated β -stable region, we draw the predicted decay modes from $Z = 90$ to $Z = 126$ in figure 3.

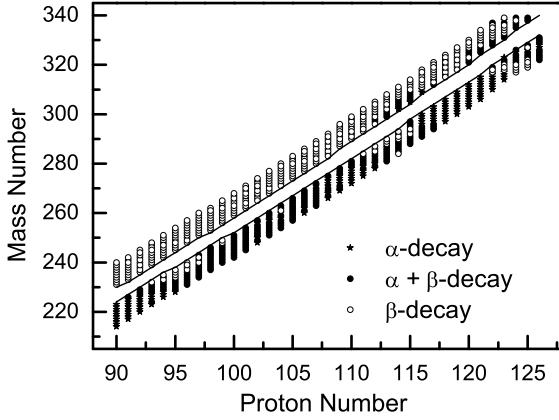


Fig. 3. The predicted decay modes of the nuclei close to the calculated β -stable region. The dark asterisks denote α -decay. The dark circles denote $\alpha + \beta$ -decay. The hollow circles denote β -decay.

In figure 3, one can clearly see the decay modes of the nuclei close to the calculated β -stable region. The dark asterisks denote α -decay. The dark circles denote α and β -decay. The hollow circles denote β -decay. The decay modes are mostly β^- -decay above the β -stable region. Especially for $Z \leq 111$, all the decay modes are β^- -decay. The decay modes are very complex below the β -stable region. All the three cases of decay mode can occur from $Z = 90$ to $Z = 126$. It indicates that the competition between α -decay and β -decay is very complex and drastic below the β -stable region. It can be seen that the nuclei above the β -stable region can occur α -decay and β^- -decay ($\alpha + \beta^-$) simultaneously when $Z \geq 112$. It is a very interesting phenomenon, because there is not the decay mode of $\alpha + \beta^-$ according to experimental results by Audi *et al.* for all Z .

4 Conclusions

In summary, we propose the β -stable region for $Z \geq 90$. The predicted β -stable nuclei in the calculated β -stable region are in good agreement with the ones obtained by Möller *et al.*. We calculate the half-lives of the nuclei close to the calculated β -stable region and systematically study the competition between α -decay and β -decay. The calculated half-lives and the suggested decay modes are in good agreement with the experimental results from Audi's Table. The predictions for half-lives and decay modes of the nuclei with $Z = 107-110$ are pre-

sented. We draw the predicted decay modes from $Z = 90$ to $Z = 126$ in a figure. We find the nuclei above the β -stable region can occur α -decay and β^- -decay ($\alpha + \beta^-$) simultaneously when $Z \geq 112$. It is a very interesting phenomenon. The competition between α -decay and β -decay is very complex and drastic below the β -stable region. The calculated results on the half-lives and the decay modes of the nuclei close to the calculated β -stable region are useful for the future experiments on heavy and superheavy nuclei.

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